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The Guest Editor Address

The 5th Workshop on Mathematical and Scientific e-Contents has its origin in the EU project Xmath (Buskerud University College 2001) focusing on the teaching of mathematics and math typesetting (MathML). Mathematics had a negative development at universities and colleges: the number of students in mathematical courses and the number passing the exams was "dangerous low". Even today, one of the main purposes of mathematical e-learning is to meet this same challenge. The scientific programme at the 5th workshop then includes topics important in this context: mathematical and scientific e-learning, markup languages, computer algebra systems, dynamic software, technologies for e-content development and educational research design. The Programme Committee is very pleased with the response from researchers into mathematical and scientific e-contents. The talks cover most of the main themes and topics of the conference from pure technology to educational research.

On behalf of the Programme Committee
Odd Bringslid (Chair).

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Case Study: Coding Theory Subject Design for Engineering Students at the University of Salamanca

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Abstract— ***Due to the special situation that we live in Spanish universities in recent years, it is necessary to change the educational system we had been using until now to a new one that includes the use of resources available on the Internet together with new technological tools and electronic devices that suppose a substantial change. The new teaching-learning process based in learning skills, also known as outcomes-learning becomes completely different from content-based learning.***

In this paper we will detail three specific tools, available for all students and useful to cover all these changes that take place in our educational system and it will be done for the particular case of the Coding Theory subject, an optional subject for the Engineers at the School of Industrial Engineering in Béjar (Salamanca, Spain).

Index Terms— ***Coding Theory, e-Learning, Matlab.***

1. *INTRODUCTION*

ENGINEERS use to work in order to find solutions to practical problems by applying their mathematical and scientific knowledge. In [10] engineering was defined as the profession in which knowledge of the mathematical and natural sciences, gained by study, experience, and practice, is applied with judgment to develop ways to use, economically, the materials and forces of nature for the benefit of mankind.

During this course the University of Salamanca will change the educational system from Technical Engineer (Europeans three-year bachelor's degrees) to a Engineering Graduate (America's four-year undergraduate degrees), a sweeping educational reform to unify the system with some European countries (Bologna Accord) [1]. Traditional education is changing to a technology based education. The teacher becomes a mediator and facilitator in the student's knowledge construction and encourages learners to question and formulate their own ideas, adjusted to the new teaching-learning processes. Until now the teaching system was centered on the content. With the Bologna accord it changed to centre on the wider competencies that the student acquires. Finally, it is important to take into account that the information and communication technologies (ICT) allow a modification in the strategies for the continuous learning of the student [8].

ICT have emerged as outstanding elements in the convergence of Universities with the common European space, known as European Area of Higher Education (EHEA) [1]. The use of computers also allows a modification of the strategies and methodologies aimed at enhancing student's life-long learning processes [4]. The technologies will thus become tools that should help us to obtain better quality teaching and, above all, they should attempt to ensure, as far as possible, that education will use methods closer to the later work activities of students and that they should endeavor to be closer to reality. The teaching of mathematics cannot be an exception to all this and should not be excluded from the use of such methods.

Learning activities are characterized by collaboration with others. Students should be able to apply what they learn at the University to the different situations that they might encounter over the course of their working lives and this could be achieved changing the learning methods to make the students build their own knowledge. During the 4 courses students have to learn how to use the computer as an instrument to solve problems and situations. In general, the use of technological tools, the management and storage of data is essential in the current collaborative way of working [5].

We will describe in this paper three more or less known tools to deal with Coding Theory subject: an online platform, specialized mathematical software, and the Internet. The final assessment consists on a public presentation of the contents of the subject with different resources chosen by the students.

This paper is organized as follows: Section 2 will deal with the use of technologies and what it implies for daily classes. In Section 3 three different tolls to learn and understand Coding

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theory subject are detailed. Finally, the conclusions will be shown in Section 4.

2. Technologies Help Teachers to change the System Education

The technologies are useful to facilitate communication between teachers and students and provide them immediate access to all types of information. In Spanish universities, both instructors and students must adapt to a new methodological change in the teaching-learning process; this will provide both groups huge advantages. Some to do with ICT have been shown to allow individualised follow-up and to increase student motivation, also improving students' results [3]. As examples, some ICT applications that we use are on-line tutorials, questionnaires, the proposal of activities, and individual contributions in which students use the Internet as a resource.

The origin of the changes promoted in the subjects we teach lies in the need to improve the actual teaching of the subjects; innovation becomes an important part of our teaching goals, with a view to facilitating the assimilation of contents by our students. Our teaching experience has shown us that the use of technological tools brings students closer to the subjects in which such resources are used. The aim of the present work is to report the results of the use of ICT in the case of Coding Theory subject and to offer an assessment of those results.

Undergraduate students are able to develop their work through innovation, and continuing improvement of processes and products by means of analytical, creative and critical thinking, together with a forward looking outlook and the ability to guide highly productive teams of workers. In their professional activities, they will have to plan, analyze and interpret, design, assess, investigate and look for possible solutions to the needs of society and those of their representative area of work or business.

The field of Mathematics is one of the most suitable for the incorporation of the technologies, since it is not only a training subject but also a scientific tool for students that should help them to solve the problems they encounter along their degree studies and in their professional careers in general. Technology programs provide a comfortable and rapid way to access, represent, and use information [6]. Some of the advantages of their use are:

- The ease with which it is possible to manage tasks and the speed with which problems can be solved, allowing students to devote more time to understanding and analyzing the results than to mechanics and the possible difficulties involved in the solution.
- Students are more eager to work in Mathematics with the aid of a computer, since this eliminates routine work and enhances creativity, increasing students' motivation.
- The technologies attract students' attention rapidly.

Additionally, ICT afford students the possibility of simulating experiments and experimenting with broadly varying situations and comparing them, something that if done "by hand" would in most cases prove very difficult, tedious or boring. They allow them, for example, to understand the real solution of a problem or the effectiveness of an algorithm by analyzing the results obtained on varying the hypotheses, initial conditions etc. Thus, the following advantages could also be mentioned:

- The graphic plotting of different curves to show the different solutions to a problem.
- Modification, suggested by the students themselves, of the data inherent to the problem and immediate visualization of the repercussions of that modification in the solution.
- The power of conviction provided by doing the calculations with a computer in the presence of the students
- Graphic plotting, in a few seconds, of counterexamples.

Thus, with the introduction of the computers and technologies into the classroom we are attempting to offer our university students a better and deeper training in mathematics, which we hope will also serve them in the future careers by adapting themselves to the advances in science and technology.

Since the advent of Information Technology, one of the main uses of the computer with respect to mathematical matters has been Numerical Calculus and Coding Theory. However, mathematical applications in engineering require not only this but also a mixture of numerical calculations and algebraic manipulations of mathematical formulae. Symbolic calculus is precisely a technology specialized in the automatic manipulation of formulae, vectors, matrices, etc. with numerical and/or symbolic elements. It is in this speciality where the use of specialized software, such as the symbolic calculus packages called Mathematica [14] or Matlab [9], is of special interest. These software packages have an easy-to-understand syntax, since the orders and commands recall the mathematical operations they execute and hence their learning is rapid and intuitive. Moreover, the help offered by these packages is very complete and is illustrated with numerous examples.

Programs such as those mentioned above are able to perform operations with real and complex numbers in precision floating comma arithmetic, operations with polynomials and rational functions of one or several variables, matricial calculus with matrices of numerical and/or symbolic elements, simple analysis (derivation, development in series, etc), the manipulation of formulae, the solving of equations (algebraic and differential), formal integration, the calculation of limits, tensor calculus, etc. The University of Salamanca has a "Campus" license for some versions of this kind of mathematical software.

The teaching of Coding Theory is not an exception and is not liable for the use of these methods [11]. The subjects of mathematics required for obtaining a degree in Industrial Engineering include, as optional subject, the Coding Theory course which will focus this studio. We are realizing the integration of eLearning as a model of further education by significantly increasing their use in recent years (Mason, 1998).

3. *Case Study: Tools for Creating a new Learning Environment for coding theory*

In engineering disciplines, web-based educational tools could help to ameliorate the deficit that could be found, sometimes, in the traditional education. Specialized knowledge can be transferred all over the world via the Internet, and new software and tools in general can be part of a comprehensive engineering higher education. In order to be able to use different software and hardware, the students need a moderate level of media competence. In addition, work-flows provided to the students must be flexible enough and be conveniently updated.

2.1 Online Platform

Virtual teaching environments are useful and represent an important part of the engineers' education. With the help of the new media, the transfer of knowledge could be much more illustrative and instructive than printed media.

However, the examination results of traditional mode students and online mode students are very similar and only some minor differences are reported in reference to results [12]. This suggests that there are no significant differences in overall performance between them.

In the case of Coding Theory subject, we use a moodle platform (called Studium in the University of Salamanca) to exchange information between lecturers and students. The virtual campus allows students to get up to date with the subject because they could access everyday in every place. The program, objectives, tasks, calendar, and information related to the subject are available on the platform. They can ask any doubt through the forums and be answered by any student or the tutor. There are also some links to download free software and to access to web pages with contents in order to keep themselves up to date.

We show two figures of the subject in the platform. In the first one (Figure 1) we can see that the students have a forum with the last news, it is also possible to send exercises and grade them. Each module of the course includes useful activities to make any topic be understood (Figure 2). In some cases we add some questionnaires and links that make available additional information about specific issues.

Figure 1: An overview of the Coding Theory subject at the web site.

For every section (see Figure 3) we upload slides showing the more important definitions, theorems and other theory results. The students can download them at home whenever they need.

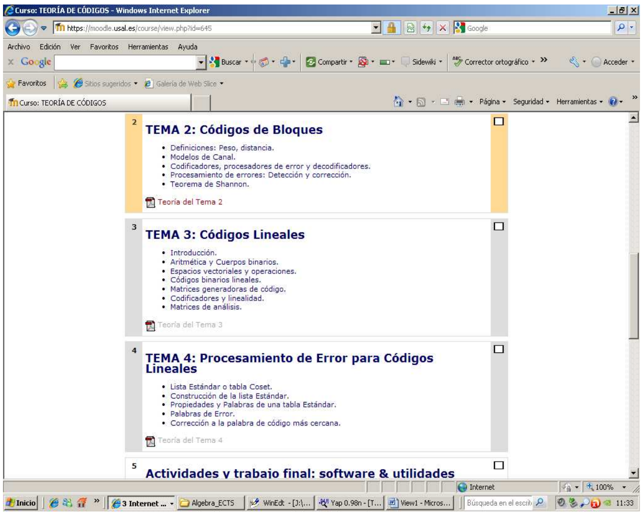

Figure 2: An overview of the contents of the Coding Theory subject in Studium.

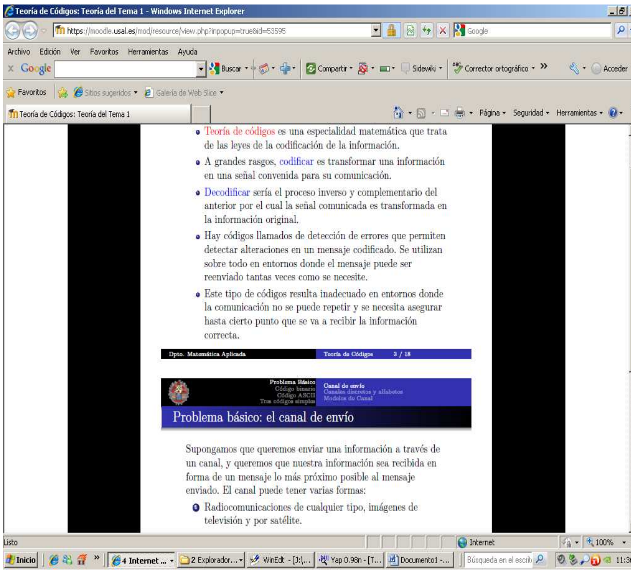

Figure 3: A pdf file added to the platform with part of the theory.

In the Programme for International Student Assessment (PISA 2000 report), among the factors that affect more directly to the school performance, the socio-educational level of students was quoted in the first place and in the second place the relationship between teachers and students was cited, therefore we consider that these kinds of tools help not only to improve the interaction among the students and between them and teachers, they are also a factor in teaching quality

2.2 Mathematical Software

The University of Salamanca has a "campus" license for some versions of some Mathematical software, like Mathematica or Matlab.

In our case, the students have chosen Matlab, because they use this software in other subjects. Moreover, Matlab includes specific functions for coding theory, to process cyclic, BCH, Hamming and Reed-Solomon codes.

We propose the students to develop some functions and procedures related to the codification process, one for each different codification system and also for error detection. In mathworks web site (http://www.mathworks.com) the relevant sections and topics that we can find in Matlab based on the coding techniques could be found.

Students start with an easy implementation of a m-file coding function, using Matlab commands like `unicode2native` that convert Unicode® characters to numeric bytes, or `dec2bin`, that convert decimal to binary number in string (Figure 4). Step by step students learn how to implement more complicated functions.

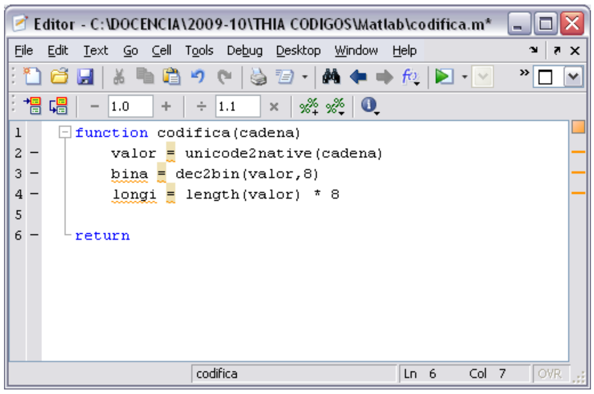

Figure 4: Implementation of an easy function with Matlab.

The advantage of using MATLAB in the subject of Coding Theory is clear if you have a look to this site: http://www.mathworks.com/help/toolbox/comm/ug/a1039545258.html which refers to the block coding techniques displayed in the "Communications Toolbox". This toolbox has the functions and objects shown in Figure 5, specifically derived for block coding.

2.3 The Internet

As we have mentioned before, one of the new methodologies useful for engineering students is technologies-based education, which refers to the learning methods that, at least, partly utilize the information and tools available through the Internet. What we propose to the students is to use online methods together with their acquired knowledge to get a more complete education in specific issues.

Block Coding Technique	Toolbox Functions and Objects
Linear block	encode, decode, gen2par, syndtable
Cyclic	encode, decode, cyclpoly, cyclgen, gen2par, syndtable
BCH	bchenc, bchdec, bchgenpoly
LDPC	fec.ldpcenc, fec.ldpcdec
Hamming	encode, decode, hammgen, gen2par, syndtable
Reed-Solomon	rsenc, rsdec, rsgenpoly, rsencof, rsdecof

Figure 5: Functions and objects that are related to each supported block coding technique.

Technology represents proactivity and progress, values that are still very much ubiquitously held, where progress is simply taken to be improvement over current local conditions. As the developers of new products and processes, engineers are a driving force for innovation in today's society. In doing their work, they rely on a large amount of information from external sources, of which the web is among the most important ones [7].

As the students, tutors are using the Internet for professional networking, learning from one another about the new media and their applications to education [13]. This skill allows the students to apply the know-how to the daily work and renew their knowledge in virtually any field of investigation.

In the case of Coding Theory subject, we proposed the students, as part of the final examination, two different activities: A research carried out using the information available at the Internet concerning barcodes and Q-R codes. In addition to this, they should look for a tool at the Internet to make an example of the subject. The students chose to do a comic trough the http://www.toondoo.com page. The student's work was based on the classes and includes part of the theory. Their works can be found specifically at http://www.toondoo.com/ViewBook.toon?bookid=206238 and at http://www.toondoo.com//ViewBook.toon?bookid=209692. An example is shown in Figure 5.

The main idea of these educational tools is to try to maintain the motivation of the student at all times, focusing his concentration on the subject without being understood as a work or a punishment.

Figure 6: One of the slides from the work done by one of the students.

It is easy to understand that one of the most important indicators of quality is the involvement of students in the school or the university, but the motivation of teachers and students alone does not imply success on mathematics or other areas, it is essential a continuous work and in this respect, we believe that a technique that may work is to keep them active with various kind of tasks.

4. Conclusion

Lecturers must help students informing and teaching about the existence, usefulness and value of the Internet and technological tools. The teaching-learning process that we have developed in recent years can change the university studies to make students to develop their own learning system.

In the particular case of the subject of Coding Theory, we have proposed the students different activities for the final assessment. The students must present an oral report of the work made during the course, summarizing the most relevant aspects. The students made a power point presentation that have included the code developed in Matlab, the comic focused on specific aspects of the subject, the particular case of barcodes and QR-codes, all of them using the online platform Studium, where they had both the manual and all the contents of theory that could need.

The final mark is focused on the documentation, presentation, and the work performed. The technical capabilities of the student were correctly tested during the questions and answers session after the presentation.

In this way, we have checked that the students need to work every day, and not only the last week of the course. They had to present an oral report made during the course, this report includes a presentation and a software program with Matlab, they also used internet to obtain new data about different codes and made a comic focused on different topics, and they used the online platform Studium. Therefore, as we have mentioned before, they have got different skills and they were able to apply all of them at the same time. Additionally we consider that this learning method is much more interactive and progressive.

Obviously, the usefulness of some of these tools depends largely on the type of course and the number and morphology of the students. As it was mentioned above, for example, the use of specialized software is much more interesting in this kind of subjects or those more related to the numerical analysis, also to perform a more continuous work and with more tasks, it is important that there are a small number of students with a similar basic knowledge.

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Mathematica and Algebra: A good marriage for the learning based on competencies

Cabello, Ana Belén; Martín; Ángel; Rodríguez, Gerardo and de la Villa, Agustín.

Abstract— ***The European Area of Higher Education (EAHE) implies a new teaching and learning model, with active methodologies and learning based on competencies [2]. We must provide our students with generic skills such as self-learning, critical thinking, team work, etc., and software packages should be used for such purposes. Thus, taking into account the above premises, the use of CAS (in our case Mathematica) could enhance the apprenticeship in this new scenario.***

We shall restrict our paper to Linear Algebra for Engineers. The use of the software includes tutorials (including general purposes of Mathematica and other tutorials focused on Linear Algebra) and examples of their use. We shall demonstrate, with examples, the different possibilities of using Mathematica. The materials will be published in a CD, part of a Spanish book addressing Linear Algebra written by one of the authors [3].

Index Terms—Bologna process, *CAS, competencies, credit, critical thinking, EAHE, evaluation process, grado, Linear Algebra, Mathematica, mathematical metacompetency, team work, self-learning, software packages.*

1. INTRODUCTION

Spanish Universities are currently in the midst of important changes. What for many countries is now the culmination of a process (to simplify the **Bologna process**, aimed at the European harmonization of University studies), in the case of Spain we are only in an initial phase. The academic year 2010-2011 is the deadline for implementing the new curricula, which will start in the first years of the "**grado**" (Spanish name for the studies corresponding to the four first University studies) of the different degree courses and should end, in six years time, with the implementation of all the postgraduate courses.

The current in-depth debate between the body of the Spanish teaching staff - as usual, hurried and at the last moment possible– is the consequence of a profound change in the learning model and is at the centre of attention of the educational process in Spain.

The Bologna model aims to put into practice a type of learning based on competencies, in which it is necessary to control the students' global work (1 credit is equivalent to 25-30 hours of student work). To achieve this, instructors must elaborate quality material and must organise students' work so that the work load will be more or less equivalent to that marked by the credits assigned to the subject in hand.

Additionally, emphasis should be placed on competencies: generic competencies (self-learning, team work, the ability to solve engineering problems, critical thinking and the use of technology) and specific competencies, which could be summarised in a single mathematical metacompetency for all engineering students: the ability to solve engineering problems using the appropriate mathematical techniques and the methods to be used (discrete or continuous, analytical or numerical, linear or not linear methods, etc). This specific competency must be reflected in each of the mathematical issues studied.

In this brain storming, the evaluation process should not be overlooked; a process that should be guided towards a continuous assessment of students' personal work, team work, self-assessment tests (provided by a LMS learning management system), small tests on concepts, and periodic controls (one per month would be a suitable proportion). Indeed any assessment´s procedure according the real possibilities (for example, the number of students per class) might benefit students' learning. They should also be offered the possibility of a final exam at the end of the academic period.

2. BASIC TEXT

Bearing these premises in minds, a standard Algebra course in an Engineering School should have contents similar to those specified below:

1. Algebra´s basic tools: Matrices. Systems of linear equations. Determinants.
2. Vector spaces: linear dependence and independence. Vector subspaces. Basis of a vector space. Dimension. Coordinates. Change of basis.
3. Linear applications: Definition and properties of linear applications. Matrices and linear applications. Eigenvalues and eigenvectors. Jordan's canonical form.

4. Euclidean spaces: Inner products. Projections. Least squares method. Orthogonal diagonalization. Orthogonal transformations.

We believe that CAS should be integrated in the teaching, since they allow students to experiment with different situations, since they do not have to make by hand heavy calculations and they can solve problems that are closer to real-life situations and not only canonical problems with the results prepared.

Their use is varied:

-By the instructor in class with different aims: demos involving graphical results, calculations, problem-solving, etc.

-Laboratory sessions in which the students can do **computer exercises** about the knowledge already gained in theoretical lectures.

-Tutorials that the students have free access to and that contain explanations of the use of the CAS in a general way or the CAS commands that they will have to use in the subject.

We shall describe our experience in the use of the CAS *Mathematica* in a course on Linear Algebra.

The course is based on the book [3], recently published (August 2010). This book contains a CD with files (different software packages DERIVE, Maxima and *Mathematica* are used) showing the different possibilities for using CAS in a Linear Algebra course.

Depending on the type of software, its use will be different since software with more features, such as *Mathematica*, demands a more rigid syntax and hence it will be necessary to employ more time for using the CAS in a fluid way.

Accordingly, the introduction of the CAS has been performed through **tutorials** through the following files:

- Tour, which analyses the possibilities of using *Mathematica* in a general way.
- General concepts that provide insight into the *Mathematica* commands useful for working with vector spaces, matrices, linear applications, determinants and sets of equations.
- Eigenvalues and eigenvectors, which explains the commands of *Mathematica* that are useful for studying the theory of eigenvalues and eigenvectors.
- Euclidean Spaces, which shows the main *Mathematica* commands related to Euclidean space and orthogonal transformations.

Remark: In the CD the names of the files are in Spanish.

Students are free to use, in their own time and under the supervision of the instructor and as often as they wish, the contents of these tutorials.

Once students believe that they are able to handle the *Mathematica* commands, they are advised to solve the problems usually solved in class manually with the help of the CAS. As an example, all the problems proposed in the text book have been solved using *Mathematica*.

The students are offered projects detailed below. We also give the detailed solution using *Mathematica*.

1. A security device has access to the images of the CCTV that focus on the four sides of a building. The device is programmed in such a way that on-screen it only shows one of the sides of the building. After showing the side of the building for one minute, it can either continue to show the image from the same CCTV, with probability *a*, $0 \leq a \leq 1$, or can access one of the two adjacent sides of the building with equal probability $\left(\frac{1-a}{2}\right)$. The security agent controlling the device introduces the value of *a* as data.
 a) Which value should s/he introduce so that the CCTV will show the same side of the building constantly? (for varying the side continuously)
 b) If $a = \frac{1}{2}$ is introduced and at 08:00 the CCTV is showing the north side of the building, determine the probability that it will show each of the sides at 09:00. What happens with other values of a?
 c) Study, as a function of *a*, the behaviour of the device when n minutes have passed, with n very large. Explain the result obtained.
 d) Perform the same study for the sides of a hexagonal building. Pay special attention to the cases $a = 0$ and $a = 1$.
 The solution and the probability distribution are shown in the figures 1 and 2.

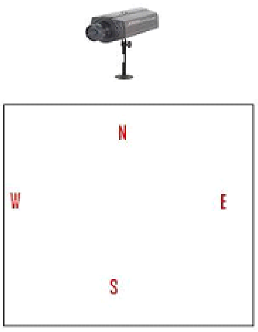


Figure 1

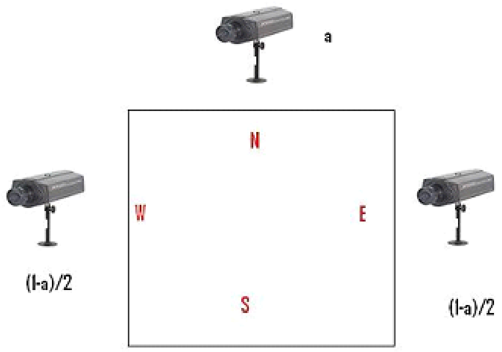


Figure 2

The probability vector for the sides (N, E, S, W) has the initial position (1, 0, 0, 0).

The transition matrix relating the states in the minutes n and (n+1) is

$$m4 = \begin{pmatrix} a & (1-a)/2 & 0 & (1-a)/2 \\ (1-a)/2 & a & (1-a)/2 & 0 \\ 0 & (1-a)/2 & a & (1-a)/2 \\ (1-a)/2 & 0 & (1-a)/2 & a \end{pmatrix}$$

The answer for a) is obvious: The first case corresponds to a =0 and the other one to the opposite case a=1. Concerning b) for a=1/2 the transition matrix is:

$$m42 = \begin{pmatrix} \frac{1}{2} & \frac{1}{4} & 0 & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{2} \end{pmatrix}$$

And the initial vector is v4 = (1, 0, 0, 0).
After one minute the position is
m42.v4= (1/2, 1/4, 0, 1/4).
After 1 hour we have to apply the process 60 times. Applying *Mathematica* command **`MatrixPower`** we get (0.25, 0.25, 0.25, 0.25)
Approximately after 1 hour there is the same probability (1/4) for showing each side.
We proceed, in similar way, for other values of parameter a. Using *Mathematica* and applying

```
N[MatrixPower[m4,n].v4]/.n➔60//
MatrixForm
```

we get

$$\begin{pmatrix} 0.25 + 0.5\,\text{a}60 + 0.25\,(-1.+2.\,\text{a})60 \\ 0.25 - 0.25\,(-1.+2.\,\text{a})60 \\ 0.25 - 0.5\,\text{a}60 + 0.25\,(-1.+2.\,\text{a})60 \\ 0.25 - 0.25\,(-1.+2.\,\text{a})60 \end{pmatrix}$$

In c) we have to study the behavior of the device, for long time, according to the different values of parameter a.

We have to distinguish the cases a=0 and a not equal to 0. In the first case the output the instruction

```
Assuming[1>a>0,Limit[MatrixPower[m4
,n].v4,n➔ Infinity]]//MatrixForm
```

gives the output `(0.25,0.25,0.25,0.25)`.
The same probability for each side happens.
The case a=0 is completely different.
The transition matrix is:

$$m40 = \begin{pmatrix} 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 \end{pmatrix}$$

The result is completely different according to the number of minutes is even or odd and it is easy to show, using *Mathematica* commands.
With n even the instruction

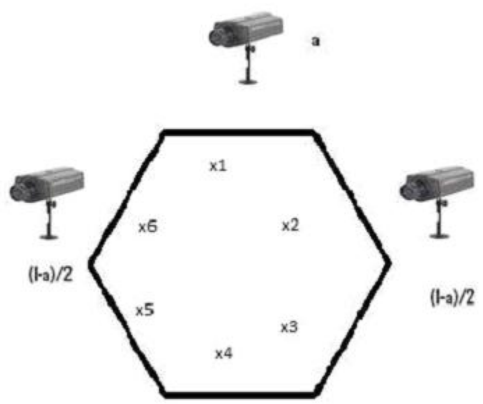
```
Assuming[Mod[n,2]=0,
Limit[MatrixPower[m40,n].v4, n➔
Infinity]]//
MatrixForm
```

gives the output `(1/2,0,1/2,0)`. The device is in the sides N or S with probability 1/2.
With n odd the instruction

```
Assuming[Mod[n,2]=1,
Limit[MatrixPower[m40,n].v4, n➔
Infinity]]//
MatrixForm
```

gives the output `(0,1/2,0,1/2)`. The device is in the sides E or W with probability 1/2.

In the case a=1 the transition matrix is the identity matrix and the device shows always the N side.
For a hexagonal building the graphical situation is:



The transition matrix is

$$m6 = \begin{pmatrix} a & \frac{1-a}{2} & 0 & 0 & 0 & \frac{1-a}{2} \\ \frac{1-a}{2} & a & \frac{1-a}{2} & 0 & 0 & 0 \\ 0 & \frac{1-a}{2} & a & \frac{1-a}{2} & 0 & 0 \\ 0 & 0 & \frac{1-a}{2} & a & \frac{1-a}{2} & 0 \\ 0 & 0 & 0 & \frac{1-a}{2} & a & \frac{1-a}{2} \\ \frac{1-a}{2} & 0 & 0 & 0 & \frac{1-a}{2} & a \end{pmatrix}$$

And we can repeat again the process and the obtained results are similar. After long time there is equal probability 1/6 for the location of the device in each side if a is not 0. When a= 0 the probability vector is (1/3,0,1/3,0,1/3,0) in the even steps and (0,1/3,0,1/3,0,1/3) in the odd steps.

Remark: A good interesting research work for advanced students could be the study of the device for an arbitrary number n of sides, paying special attention to the case a=0 if n is odd or even. The result is completely different.

2. The distribution of the traffic in a street network is as follows (see figure 3)
   The directions are indicated by the arrows. The only two-way streets are AB and CD Find the traffic flow in each stretch in the following cases:
   i) At node C there are road works and we want the traffic to be minimum.
   ii) Circulation through node E is prohibited.
   iii) Circulation in the AB stretch and the D node is prohibited.

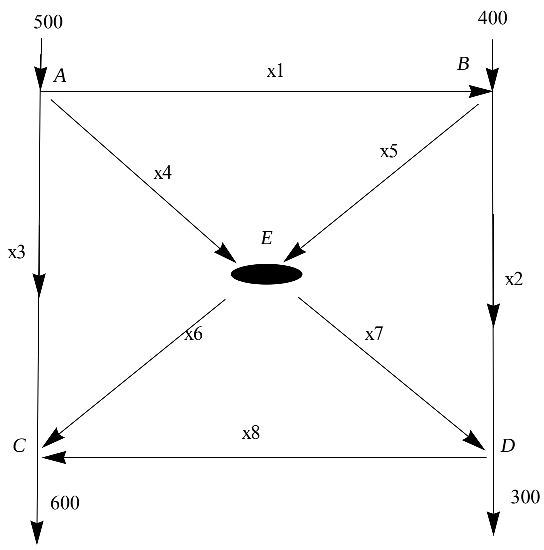


Figure 3

For solving the problem we use the *Mathematica* command Solve. This command allows to solve systems of linear equations.
First of all we model the problem.
x1 and x8 can be positive or negatives
In the node C the traffic is constant (600). Then the sum of all entries must be 600.
According the traffic in each node the system of linear equations corresponding to the traffic is:
Node A: 500 = x1 + x4 + x3
Node B: 400 = x2 + x5 - x1
Node E: 0 = x4 + x5 - x6 - x7
Node C: 600 = x3 + x6 + x8
Node D: 300 = x2 + x7 - x8

Solving the system:

```
Solve[{500==x1+x4+x3, 400== x2 +
x5 - x1, 0== x4 + x5 - x6 - x7,
600== x3 + x6 + x8, 300 = x2 + x7
- x8},{x1,x2,x3,x4,x5,x6,x7,x8}]
```

The output is

```
x1=-100+x5-x7+x8
x2=300-x7+x8
x3=600-x6-x8
x4=-x5+x6+x7
```

The system has infinite solutions. In the node C the number of entries is:

x3+x6+x8/.{x3 ➔600-x6-x8,x6 ➔x6, x8➔x8}

And the result is, obviously 600.

For solving the question ii) we assume the traffic in E is 0. Then x4 = x5 = x6 = x7 = 0**.**
We solve the system after introducing the new equations.

```
Solve[{500==x1+x4+x3, 400== x2 +
x5 - x1, 0== x4 + x5 - x6 - x7,
600== x3 + x6 + x8, 300 = x2 + x7
- x8,x4==0, x5==0, x6==0,
x7==0},{x1,x2,x3,x4,x5,x6,x7,x8}]
```

The corresponding output is

```
x1=-100 +x8
x2=300+x8
x3=600-x8
x4=0
x5=0
x6=0
x7=0
```

x2 and x3 must be positive and the values of x8 are in the interval [-300,600]. We find the extreme values corresponding to x8=-300 and x8=600.

The obtained results are:
If x8=-300 then x1=-400, x2=0 and x3=900.
If x8= 600 then x1= 500, x2=900 and x3=0.

For the question iii) we have:

x1 = 0 (the traffic is forbidden in AB).
x2 = x7 = x8 = 0 (the traffic is forbidden in the node D).
Solving the new system

```
Solve[{500==x1+x4+x3, 400== x2 +
x5 - x1, 0== x4 + x5 - x6 - x7,
600== x3 + x6 + x8, 300 = x2 + x7
- x8, x1==0, x2==0, x7==0,
x8==0},{x1,x2,x3,x4,x5,x6,x7,x8}]
```

The result is {}
The system is inconsistent. But the result is obvious because one equation is
300 = x2 + x7 - x8 and it is impossible with the conditions: x2 = x7 = x8 = 0.

Additionally, the use of CAS propitiates the solving of real problems in the engineering world. Thus, the students are invited to find this type of problem where it is necessary: to solve sets of equations with a large number of unknowns; the diagonalization of matrices or the computation of eigenvalues or eigenvectors, approximation via the least squares procedure, etc. To accomplish these aims, they are supplied with the commands or procedures that will allow them to use the most appropriate techniques to solve sets of equations; for example, the LU and Cholesky methods, and the QR decomposition for the factorization of a matrix as a product or a triangular and orthogonal matrices. Also implemented, for student use, are the Jacobi and Gauss-Seidel methods for the numerical resolution of sets of equations and the power method for the calculation of eigenvalues and eigenvectors. Some procedures have been based in [4].

Finally, it is appropriate to foster a critical attitude in the students and to ensure that they do not view CAS as a “black box”, about which they wish to know nothing except that it provides the required results. Accordingly, they should be taught that the outputs of CAS must be analysed and that it they, themselves, who must control the CAS and not vice-versa. Examples of mistaken, unexpected, or impossible results should lead students to a balanced use of CAS, always controlling the process [1].

3. CONCLUSION

CAS can be of help in the adaptation of University studied to the Bologna process through instruction based on competencies, in which self-learning, a critical attitude and problem-solving (in our case, engineering problems) are an important part of the educational process. The use of CAS allows real and not only “didactic” problems to be posed.
The possibility of experimenting, changing equations, parameters, etc. can help to understand the issues in hand.

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Computer-assisted environments as predictors for higher cognitive learning activities in physics

Urban-Woldron, H.

Abstract—***We studied individual as well as psychological factors which affect students' learning motivation and cognitive learning activities in a computer-assisted environment. Computer-based tools, virtual experiments and simulations were used to facilitate learning. Nine physics teachers of Austrian secondary and high schools participated in a professional development course for integration of technology and digital media into the classroom and used computer-assisted teaching materials. A questionnaire was used to measure students' motivational orientations, their use of different learning strategies and their perception of the technology-enriched environment. A confirmatory factor analysis using AMOS was employed to test the assumed causal relationships in the research model. The results show the importance of students' experience of competence directly affecting both learning motivation and the development of higher cognitive learning activities.***

Index Terms— ***Cognitive learning activities, experiencing competence, learning motivation, perception of the technology-enriched teaching-learning environment***

1. *INTRODUCTION*

RECENT reviews of the effects of ICT (Information & Communication Technologies) in science lessons show that teachers do not yet exploit the creative potential of ICT and do not engage students enough in the production of knowledge [2]. Therefore teachers need training and continuing professional development in the use of ICT in order to carefully integrate ICT into the teaching process and to provide appropriate guidance [7,21]. Actually, ICT-rich environments already provide a range of affordances to enable learning of science. But researchers suggest that integrating these affordances with other pedagogical innovations provides even greater potential for enhancement of students' learning [3]. Beyond that, one of the most important things to understand about technologies is that particular technologies have both specific properties that allow certain actions to be performed encouraging specific types of learner behaviour but also have their constraints [22]. Therefore, the thoughtful pedagogical uses of technology require the development of a complex, situated form of knowledge that Koehler & Mishra call TPACK (Technological Pedagogical and Content Knowledge) [10]. Shulman's idea of Pedagogical Content Knowledge (PCK) [19] for bridging most effectively the gap between students and specific content, e.g. developing a deep understanding of the content, is extended to the domain of technology. As illustrated in Fig. 1, TPACK is settled at the intersection of all three relevant elements of teacher knowledge, which cover content knowledge, pedagogical knowledge and technological knowledge.

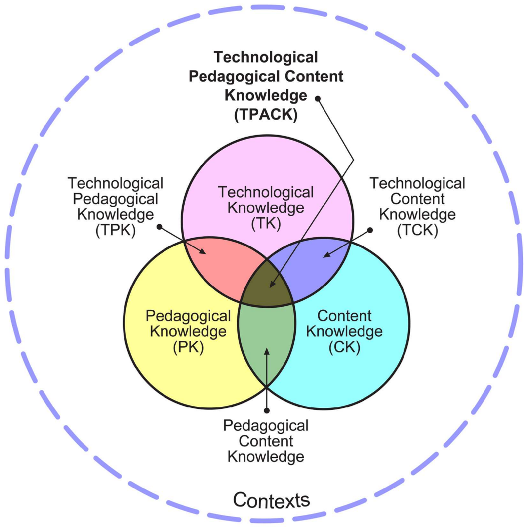

Figure 1. TPACK framework for teacher knowledge[1]

In conclusion, effective technology integration has to bring together the teaching of subject matter with effective and appropriate uses of technology accompanied by the refinement of teaching practices to engage students in the use of technology as they investigate curriculum, express what they know and understand, and apply knowledge to construct new meaning [8]. Naturally, to meet these needs, sensible technology integration certainly requires teachers' development of insightful sensitivity to the dynamic, transactional relationship between all three components of teacher knowledge [13].

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[1] Source: http://tpack.org/tpck/images/tpck/a/a1/Tpack-contexts.jpg (accessed on October 12, 2010)

According to Archambault and Crippen [1] the TPACK framework is a useful organizational structure for defining what it is what teachers need to know to be able to integrate technology effectively. However, an essential question concerning issues about technology integration lies in (1) how teachers can learn to infuse technology innovatively into subject area instruction and learning and (2) how to help teachers to make individual meaning of new constructs and experiences with technology to determine its impact on education, including learning processes, access to content and instructional methods [14]. According to the conceptual framework of Dywer, Ringstaff and Sandholtz [18], teachers have to move through an evolution of thought and practice (ETP) when learning to use technology in the learning process.

The present study is based on the findings of extensive research in the field of technology integration and learning motivation and it explores the relationships between student characteristics and student outcomes concerning their learning motivation and learning activities. With regards to contents, the study covers students' perception of kinematics graphs. The learning activities used in the study were inspired by publications uncovering a consistent set of student difficulties with graphs of position, velocity and acceleration versus time. In an early study, Thornton and Sokoloff [20] found that students using real-time graphs showed higher improvement of their kinematics graphing skills and their understanding of the qualitative aspects of motion they observed, compared to students using delay-time graphs. For developing the conceptual understanding and reasoning skills necessary to teach science as a process of inquiry, materials developed by McDermott [11,12] were used after adapting them to the particular student age.

Computer-based tools, virtual experiments and simulations were used to facilitate student learning in physics. The laboratory activities made use of motion detectors in combination with computer interfaces and interactive software tools to allow students to collect relevant data and to perform a series of analyses. Nine physics teachers of Austrian secondary and high schools who volunteered for participating in the study used provided computer-assisted teaching materials in the physics classroom.

Based on the conceptual frameworks of TPACK and ETP, a blended learning course for teacher training, including a half-day session, was designed and offered to the teachers carrying out eLearning-projects granted by the IMST[2] Fund. The course was held over 10 months and supported teachers in implementing the use of a motion detector in their classrooms. It started with a face-to-face session which was attended by all nine teachers, all being novices in using motion detectors in the physics classroom. session, the teachers were supported by an electronic platform, where they were expected to discuss their lesson plans and collectively reflect on their teaching activities and how to become more learner-centred when implementing technology in teaching kinematics.

2. Research Questions

The theoretical basis of the study is Deci and Ryan's [4] self-determination theory, as well as Eccles and Wigfield's [5] expectancy-value theory, applied to ICT-enriched teaching-learning environments. Research has shown that a positive perception of supportive learning conditions is strongly related to self-determined learning motivation, to interest development and to the application of deep learning strategies [17]. According to the self-determination theory, the individual perception of autonomy and competence support within the ICT-rich environment is assumed to enhance self-determined learning motivation as well as a deep processing of learning contents. The influencing factors shown in the causal model (see Fig. 2) include a group of exogenous personality variables derived from theory: Expectancy of content-specific self-efficacy, content task value and endurance and willingness to achieve. The variables ICT-rich learning environment, learning motivation and cognitive learning activities are endogenous, whereupon the first two mentioned are viewed as intermediate variables in the model.

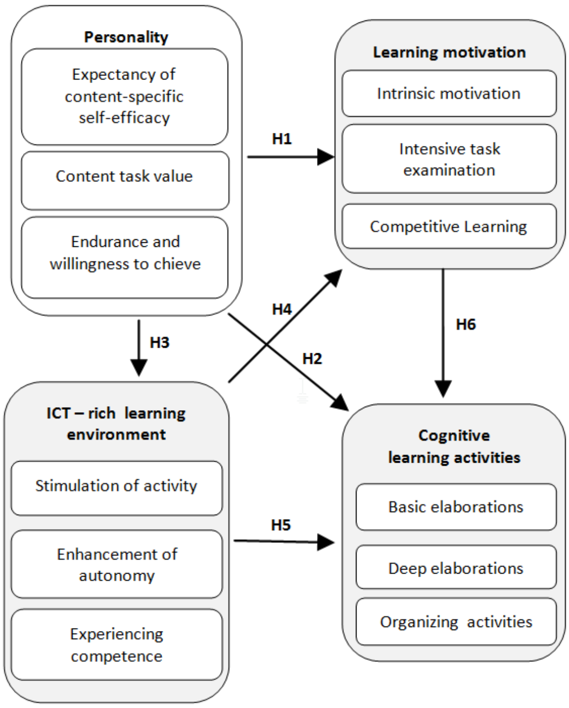

Figure 2. Research Model

Specifically, the following two research questions were raised:

(1) To what extent are students' characteristics related to how they self-regulate their learning in the ICT-rich environment?

(2) What is the impact of the ICT-rich environment on students' learning outcomes?

[2] IMST (Innovations Make Schools Top http://imst.uni-klu.ac.at)

Based on Fig. 2, the hypotheses of this study are described in Tab. 1.

H1	Student characteristics have positive effects on learning motivation
H2	Student characteristics have positive effects on cognitive learning motivation
H3	Student characteristics have positive effects on the perception of support through the ICT-rich environment
H4	Perception of the supportive learning conditions in the ICT-rich environment directly affects learning motivation
H5	Perception of the supportive learning conditions in the ICT-rich environment positively effects cognitive learning activities
H6	Learning motivation has a positive effect on the implementation of higher cognitive learning activities

Table 1. Hypotheses of the study

3. Methods and Samples

Learning motivation is supposed to be a relevant concept for self-controlled learning in a computer supported learning environment. In the study the quality of the learning environment is measured by the quality of learning motivation and learning activities reported by the students. As a theoretical framework for developing the student questionnaire, the self determination theory and the theory of interest [4,9] were used.

It is assumed that the quality of learning motivation and cognitive learning activities is significantly associated with perceived support of autonomy and competence as well as with the stimulation of activity.

The student survey form administered in this study consisted of two parts. The first part contained items gathering students' demographic information. The second part comprised sets of items measuring students' task value, self-efficacy, persistence and achievement goals, learning motivation and cognitive engagement and perception of the technology-enriched environment. Whereas the items for assessing learning motivation (QLM) and learning activities (CLA) were primarily drawn from literature [15,16] and accordingly adapted, the items concerning the impact of educational technology (SA, EC, EA) were newly developed, building on findings of the relevant research literature. Some descriptions for the measures and their Cronbach's alpha internal consistencies derived from the present sample are provided in Tab. 2.

Scale	Items	α	Sample item
Learning motivation (LM)	11	0.87	"I liked the assignments, because they simulated me and aroused my curiosity"
Cognitive learning activities (CLA)	11	0,91	"The assignments provoked me to activate my prior knowledge"
Stimulation of activity (SA)	11	0.84	"The assignments inspired me to investigate them on my own."
Experiencing competence (EC)	10	0.87	"The assignments enabled working independently and play around with ideas of my own."
Enhancement of autonomy (EA)	11	0.81	"I could autonomously direct my activities within the assignments".

Table 2. Description of measures

To evaluate personality variables of the students, a questionnaire to gather information about (1) content specific self-efficacy expectancy, (2) content task value and (3) persistence in physics tasks of the students was used (see Tab. 3).

Personality	M	SD	α	Items	skewness
Expectancy of content specific self-efficacy (ECS)	2.48	0.72	0.81	4	-0.08
Content task value (CTV)	2.83	0.73	0.87	7	-0.30
Endurance and willingness to achieve (EWA)	2.66	0.80	0.84	5	-0.11

Note: Scales 1 = completely disagree, 4 = completely agree

Table 3. Descriptive statistics of personality dimensions

The items comprising the scale *content specific self-efficacy expectancy* (ECS) assess aspects of students' self-appraisal of their ability to master a physics task including judgements about their confidence in their skills to perform that task (e.g. "*I was confident that I could master the physics assignments*"). The items constituting the scale *content task value* refer to the students' evaluation of how interesting, how useful and how important the physics task is (e.g. "*Understanding the physics concepts in this class is very important for me*"). The scale 'persistence in physics tasks' aimed at assessing students' ability to maintain their learning intentions referring to individuals' ability to stay within study activities even when the tasks are difficult or there is lack of interest (e.g. "*In general, if I have to solve a problem and I cannot manage that at first try, I try as often as necessary until I succeed to solve it correctly, or at least, I do not give up until I have tried many times and in very different ways*").

Constructs	Dimensions	Items	α	%([3])
Students' higher order learning processes				
Quality of learning motivation	Intrinsic Motivation	5	0.88	72.8
	Intensive task examination	3	0.80	
	Competitive Learning	3	0.87	
Cognitive learning activities	Basic elaborations	3	0.84	72.6
	Deep elaborations	3	0.81	
	Organizing activities	5	0.83	
Students' perceptions of supportive learning conditions within the ICT – enriched learning environment				
Enhancement of autonomy (EA)	Self-evaluation	3	0.77	72.0
	Implementation of own ideas	6	0.80	
	Self-organization	2	0.74	
Experiencing competence (EC)	Encouragement of understanding	2	0.76	75.5
	Supporting learning processes	4	0.78	
	Working independently	4	0.80	
Stimulation of activity (SA)	Learning becomes interesting	2	0.80	79.1
	Stimulating engagement	4	0.76	
	Allowing exploration	5	0.77	

Table 4. Validities and reliabilities of constructs in the model

The statements for the constructs and dimensions displayed in Tab. 4 were designed to measure students' perception of learning motivation, cognitive learning activities as well as supportive learning conditions within the ICT-rich environment. Factor analyses with the principal-components method and varimax rotation were employed to identify and extract factor dimensions. Tab. 4 shows satisfying reliabilities and cumulative percentages of variances of the subscales used in the model.

Tab. 5 provides an overview on correlations of all sample variables.

[3] Variance Explained

		1	2	3	4	5	6	7	8
Personality (Expectancy of content specific self-efficacy / ECS, content task value / CTV & endurance and willingness to achieve / EWA)									
1	ECS	–							
2	CTV	,559**	–						
3	EWA	,456**	,600**	–					
Quality of learning (Learning motivation / LM & Cognitive learning activities / CLA)									
4	LM	,604**	,765**	,689**	–				
5	CLA	,514**	,680**	,671**	,719**	–			
Perception of the ICT – enriched learning environment (Enhancement of autonomy / EA, stimulation of activity / SA & experiencing competence / EC)									
6	EA	,431**	,650**	,747**	,678**	,671**	–		
7	SA	,451**	,644**	,752**	,696**	,653**	,806**	–	
8	EC	,465**	,662**	,806**	,712**	,751**	,824**	,833**	–

*Note: ** p < 0.01; relevant correlations above 0.6 are printed bold*

Table 5. Pearson correlation matrix of all relevant variables

A sample of 299 students out of 14 classes, which were educated by the nine teachers who participated in the professional development teacher training course provided complete data, which were used for analysis. The students were from grade 6 to 10 and had a mean age of 14.1 years. 165 students were male, 143 were female; 34.5% of the students attended a middle school, 31.1% a secondary lower and 25.4% a secondary upper school and 9% a vocational school.

The online questionnaires were completed under the supervision of the responsible teacher. The items were aligned on a Likert scale, ranging from 0, "*I totally disagree*" to 4, "*I totally agree*". Besides descriptive statistics, structural equation modelling was used to test the relations between the perceived technology-enriched environment and the quality of learning motivation and higher cognitive learning activities.

4. *Results*

Based on the research model shown in Fig. 2, confirmatory factor analyses using AMOS were conducted to test the causal relationships in the model.

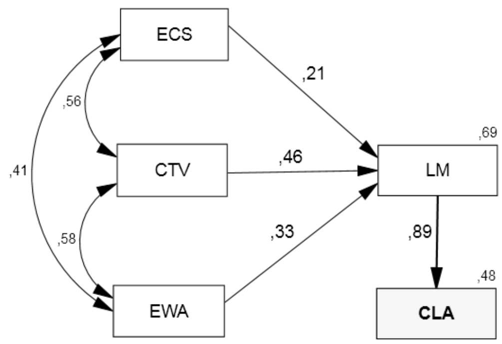

Figure 3. Results of AMOS analysis for model 1

The results for model 1 testing H1 and H2 indicate that the personality variables account for 69% of the variance of learning motivation and 48% of the cognitive learning activities and that the most relevant predictor for learning motivation is the content task value. None of the three personality variables has a direct effect on cognitive learning activities. The indices for the fit of the measurement model 1 advocate for a very good fit: χ^2 = 0.987, DF = 2, p = 0.611, CFI = 1.000, RMSEA = 0.000, PCLOSE = 0.782.

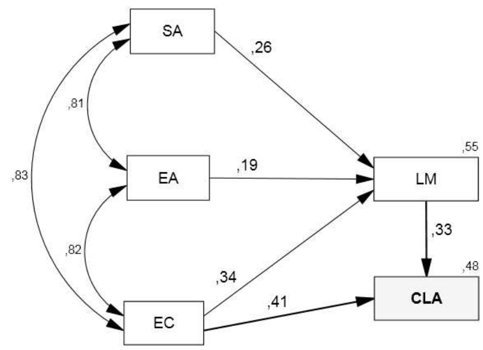

Figure 4. Results of AMOS analysis for model 2

The results for model 2 testing H4 indicate that the perception of the computer-assisted environment accounts for 55% of the variance of learning motivation and 48% of the cognitive learning activities and that the most relevant predictor for learning motivation and cognitive learning activities is the support of students' competence. Stimulation of activity and enhancement of autonomy do not directly influence the occurrence of higher cognitive learning activities. The indices for the fit of the measurement model 2 as well advocate for a very good fit: χ^2 = 0.664, DF = 2, p = 0.717, CFI = 1.000, RMSEA = 0.000, PCLOSE = 0.850.

Incorporating all variables in model 3 (see Fig. 5) it appears that the variables SA, EA and EC, which apply to students' perception of the supportive conditions within the ICT-rich environment, become quasi intermediating variables and highlight the indirect effects of the students' characteristics on the implementation of cognitive learning activities. Students with high content task values and achievement goals feel strongly motivated through the ICT-rich environment and in consequence perceive support of autonomy and competence and finally perform higher cognitive learning activities.

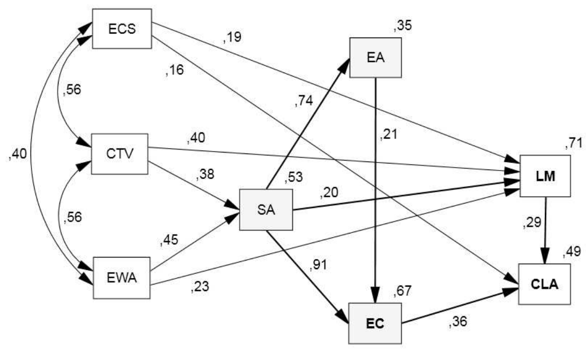

Figure 5. Results of AMOS analysis for model 3

The standardized total effects between all variables in the model are shown in Tab. 6. A few relevant values will be briefly discussed: First, stimulation of activity seems to represent the most relevant predictor for the implementation of higher cognitive learning activities (β = 0.45) as being a component of various paths (e. g. CTV$\rightarrow$SA$\rightarrow$EC$\rightarrow$CLA or CTV$\rightarrow$SA$\rightarrow$LM$\rightarrow$CLA).

Second, enhancement of autonomy practically does not significantly affect students' learning as it plays the role of a mediating variable directly affecting the perceived support of competence.

	CTV	EWA	SA	EA	ECS	EC	LM
SA	,38	,45	,000	,00	,00	,00	,00
EA	,28	,33	,74	,00	,00	,00	,00
EC	,40	,48	1,00	,21	,00	,00	,00
LM	,48	,32	,20	,00	,19	,00	,00
CLA	,28	,27	**,45**	**,08**	,22	**,36**	,29

Table 6. Standardized total effects (model 3)

To sum up the above results, learning motivation is chiefly determined by personality variables of the students, whereas the perceived support within the ICT-rich environment directly affects the emergence of deeper learning processes. The standardized indirect effects for personality on cognitive learning activities in model 1 through learning motivation range between 0.19 for ECS and .41 for CTV. Therefore the hypotheses H1, H3 and H6 were confirmed, and the hypotheses H4 and H5 were partly validated. The hypothesis H2 has to be rejected, as in all three models the students' characteristics, emphasizing content task value, expectancy components of self-efficacy for learning and performance as well as effort regulation, do not directly affect higher cognitive learning strategies.

5. Conclusion

Although the findings of the present study relied on data yielded from self-report surveys of students, important implications for both research and practice can be derived. First, taking a look at students' outcomes, it appears reasonable to embed technology in a way that it stimulates interactivity and enhances support of autonomy and competence. Second, as the motivational regulation of student learning is mainly supported by student characteristics, it can be assumed that effectively integrated technology into physics content significantly augments the cognitively engagement in learning tasks. Third, therefore it is likely that TPACK of the teacher may be a significant predictor of how students perceive the ICT-rich environment and how they engage in higher cognitive learning activities. Further research should focus on professional development of teachers' TPACK, making them capable of using technologies in constructive ways to teach content, focusing on conceptual understanding and self-regulated learning of their students.

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An Effective Use of CAS for Reasoning as a Cognitive Tool

Takahashi Tadashi

Abstract—*The effective use of CAS (computer algebra system) were introduced and were expected to develop ways in which to use CAS effectively as "tools" for mathematics education and to achieve good results as a teaching method for mathematics education. In the 2000s, with the evolution of software and hardware, it has become easy to use the CAS in classroom. However, the proportion of teachers who use CAS in classroom is still quite low and it is hard to say that CAS has started to be commonly used in classroom. We must consider about the effective use seriously. Then we need to consider "a tool theory" in the cognitive science. Humans use strategies to solve problems. Strategies are used as knowledge to plan solutions and decide procedures. The techniques for theorem prove using CAS is being developed. We must consider the theorem prove from not only the perspective of its effect on cognitive science, but also from the perspective of mathematical studies. We can explain new use possibility of the CAS by being based on the theory of cognitive science.*

Index Terms—*Mathematics education, Computer algebra system, Cognitive science*

1. *INTRODUCTION*

In the 1990s, there were introduced computer -based mathematics education, and the effective use of CAS (computer algebra system) was part of this attempt. These proposals were expected to develop ways in which to use CAS effectively as "tools" for mathematics education and to achieve good results as a teaching method for mathematics education. However, these efforts lacked a clearly defined direction. Researchers were uncertain as to what kind of basic principles the utilization of CAS for mathematics education stood for, or what goals we were trying to achieve.

In the 2000s, with the evolution of software and hardware, it has become easy to use the CAS in classes. However, the proportion of teachers who use CAS in classroom is still quite low and it is hard to say that CAS has started to be commonly used in classroom. There is the fact that the value of CAS as a teaching tool is not recognized.

In mathematics education, what is the purpose of using CAS? That is to assisting the developing of students' mathematical thinking. In particular, the CAS has big possibility in learning of the mathematics. However, we did not perform a clear study about its possibility. It is because we thought the CAS to be the expert engineers' tool. The present condition has changed. We must consider about the effective use seriously. Then we need to consider "a tool theory" in the cognitive science. We can explain new use possibility of the CAS by being based on the theory of cognitive science.

According to the three-level human behavior model of Rasmussen, automatic human actions can be classified into the three levels of skill-, rule- and knowledge-based actions(Fig.2.1, [9]).

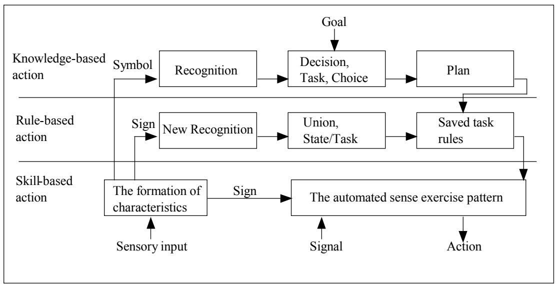

Figure 2.1 Three-Level Model of Human Action

A skill-based action is a response that occurs in less than 1 second ([7]). A chain of skill-based actions is a rule-based action. Thinking about how to solve a problem is a knowledge-based action.

Skill-based actions are performed smoothly without intentional control. Rule-based actions require a great deal of repetitive practice in order to be transferred to the skill-based level. First, the external conditions must be recognized, then the rules for composing the act are combined with the conditions required to carry out the behavior. Knowledge-based actions require the recognition of external conditions, the interpretation of these conditions, the construction of a psychological model for considering solutions, planning, and finally, the use of the other two behavior levels to carry out the action.

This is a process model in which mastery of behavior requiring thought is internalized to the point where it can be carried out unconsciously. Mistakes can be explained as omitted steps, or for example, as pushing the wrong nearby button in smoothly carried out skill-based actions. In the case of knowledge-based actions, illusion can lead to error. In the present study, this process was analyzed using Rasmussen's three-level human behavior model in order to identify what functions are essential to facilitating smooth

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action and learning. Behavior used to learn about problems and how to solve them is classified in detail according to the three-level model. Humans act by classifying issues and their relationships by consciously combining them. Humans control themselves by constantly observing, thinking about, evaluating, and integrating their behavior in order to achieve accuracy, continuity, consistency, and normality ([4]).

2. Cognitive Science in Learning

2.1. Norman's theory

CAS can be used as effective tools for calculation and mathematical thinking. CAS is a kind of external tool. An external tool can help a person promote the use of internal tools and thus improve the efficiency of overall use of internal and external tools ([6]). The effective use of CAS can play an important role when a person is trying to understand new concepts. As an external tool, CAS was developed to actually save a lot of time and effort in making calculations and enable faster manipulation of mathematical formulae. Although modern CAS is named algebra systems, it actually has much more performances than symbolic manipulation. With these performances, CAS has become even more powerful tools for mathematics. If we hope to utilize CAS in mathematics education, we need to find some new methods of teaching (based on the accomplishments obtained from cognitive science) that differ from the traditional methods of using "paper and pencil".

Careful consideration of the aspects of the science as an "external tool" is important for bringing about changes in approaches to the problem. CAS as the external tools substitute for some human abilities and thus bring a new potential. At the same time, CAS requires different kinds of abilities and knowledge. When we consider the effects of mathematics education utilizing CAS, clarifying what kinds of abilities and knowledge are required is crucial.

A cognitive tool is a tool for embodying the image of an outer object that appears in the consciousness on the basis of the human perception. Norman divided the human cognition, when using technology, into two categories ([5]).

- Experimental cognition: to cope without conscious to the outside world change.
- Reflective cognition: to deeply understand for thinking of the meaning of each thing and referring back to experience.

In a learning environment using CAS, the human cognition is divided into experiential one and introspective one. The tools for the experiential cognition have to be able to exploit a rich sensory stimulus. The tools for introspective cognition must be supporting the search of ideas. This cognition needs a different support. It does not function well if we give experiential cognition an introspective tool. Vice versa also will not work. Therefore, teachers must distinguish which type of cognition the tools are supporting. And for that purpose, the teachers must provide tools that offer appropriate support for a certain activity. In order to use effectively technology, the teachers must consider which one of the two cognitions is suitable for its learning activity. If the consideration for the cognition mode is neglected, the effective use is impossible. If learners enjoy the experiences when they must do introspective searches, they misunderstand its activity as introspective activity.

According to Norman, there are three learning categories that are useful in determining which of the two cognition modes is suitable for its learning activity:

- Accretion: to accumulate facts.
- Tuning: to adjust it in a way to use skills involving introspection with an experiential mode.
- Restructuring: to form an appropriate conceptual structure by introspection.

In many cases, accretion and tuning are seen as experiential modes, restructuring is seen as an introspective mode. To make CAS appropriately work as a cognitive tool, teachers must determine what responds to a certain learning activity of students.

2.2. Strategies

Strategies are used as knowledge to plan solutions and decide procedures. When these procedures, in general or for the most part, obtain the correct answer, the procedure is called a heuristic; however, such heuristics do not always result in a correct solution.

Strategies are used even when human beings solve mathematical problems. Recognition knowledge and experience are used as "doing it like this is effective in this case". The ability to rapidly reference knowledge is required for strategies based on experience. The famous book by the mathematician Polya, "How to solve it"([8]), showed the processes of mathematical problem solving; however, one can not learn how to use heuristics in problem solving just by reading a book.

In researching problem solving, there are two contrasting concepts. The first emphasizes insight, flash, and senses, while the second emphasizes experiential knowledge. The former concept employs a strong tendency to perceive that strategies of thought are learned through the experience of problem solving. In other words, it is assumed that an intuitive feelings and specific technical abilities can be acquired. In the latter concept, it is assumed that problem solving ability arises from the accumulation of rules inherent to the domain provided by an individual problem.

Such differences depend on the problem's nature, domain, and level, and the type of person involved in the learning process. In addition, it is difficult to establish clear boundary lines between these two concepts. In problem solving, experiential knowledge plays a large role. Heuristics are general ideas or algorithms (a procedure providing the correct solution), and are widely used. Heuristics are equal to "the logic of a thought".

Examples of extremely general strategies are "try to draw a figure if you come across a difficult problem", and "search for similar problems that you have experience with". There are also concrete strategies we are familiar with, such as "A problem requiring the comparison of quantities requires two differences, and a transform formula" and "try to make clauses that differ next to each other for number sum sequence problems" ([2]).

3. *The Effect of the Theorem Prover*

As a representative of a theorem prover, the Isabelle/HOL system was used. Research on formalizing abstract algebra in Isabele/HOL is based on work by Hidetsune Kobayashi. This study focuses on researching mathematics, and in particular, on training researchers in the techniques of proving ([3]). In the area of mechanical theorem proving, Kobayashi gave a decision procedure for what he called abstract algebra, based on algebraic method. It is really surprise to prove many abstract algebra theorems whose traditional proofs need enormous amounts of human intelligence.

One of the key observations of Kobayashi is that theorems in abstract algebra can be relatively easily dealt with by a lot of lemmata, completely from former methods.

The power of the method can be shown by experiments on computers in which many abstract algebra theorems were proved. The success of Kobayashi's method stimulated researchers to apply the connection of lemmata images. This research on formalizing abstract algebra in Isabelle/HOL is being conducted in order to develop a CAS that supports mathematical study focused on “abstract algebra”. The system combines methods of automated theorem proving and also integrates programming in a natural way.

This method is of interest to researchers both in artificial intelligence (AI) and in algebraic modeling because they have been used in the design of programs that, in effect, can prove or disprove conjectured relationships between, or theorems about, abstract algebraic objects. It is interesting to note that theorems have been verified by this method. In a limited sense, this "theorem prover" is capable of "reasoning" about algebraic conjectures, an area often considered to be solely the domain of human intelligence.

This research aims at extending current computer systems using facilities for supporting mathematical proving. The system consists of a general higher-order predicate logic prover and a collection of special provers. The individual provers imitate the proof style of human mathematicians and produce human-readable proofs in natural language presented in nested cells. The long-term goal of this research is to produce a complete system, which supports mathematicians. On the meta-level, we can write explicit programs for reasoning tactics using Isabelle/HOL.

When researchers use the theorem prover for the acquisition of knowledge or skills, we must consider a "tool" to be a "symbol device". A symbol device exists between the researchers and the research subject. Operation activity occurs between a symbol device and the researching subject. In cognitive science, two difficulties exist, one in the interaction between the researcher and the symbol device, and one in the interaction between the symbol device and the research subject. Therefore, we must overcome these difficulties in order to effectively utilize the theorem prover in cognitive science. Moreover, we must assess the benefits of considering the integration of the theorem prover from the perspective of the relationship between mathematical knowledge and mathematical concepts. When theorem provers are used in mathematical studies, researchers achieve a result through their efforts. Then, the researchers must investigate whether conceptual problems exist or whether they simply do not appreciate how the theorem prover works. By using a theorem prover effectively, researchers become aware of numerous mathematical ideas. This is made possible by incorporating the results of research in cognitive science. In carrying out a seven-phase model of human action, "the formation of a series of intentions or actions" must be performed smoothly. The effective use of a theorem prover in cognitive science is influenced by the contents of mathematical thought, and research and understanding of mathematics can further influence general idea formation. The theorem prover influences the “perception - interpretation – evaluation” phases of evaluation. The foundations of this model were studied by Rasmussen as the three-level control model of individuals’ actions ([10]).

We can use the theorem prover as a material object that is available for the assessment of human activity. The use of the theorem prover can establish automatic and routine procedures. Controlling this automation is essential, especially in research on though processes. There are three methods for creating a theorem proof (by hand, by mind, and with a computer). A researcher's point of view of cognitive science considers the relationship between the brain and mind as the relationship between hardware and software in a computer. According to this point of view, the science of the mind is a special science, the science of thought.

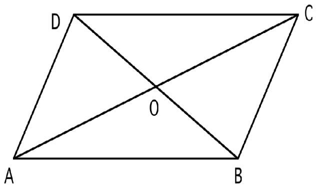
We introduce an application of CAS in Theorem proving([1]). Theorem proving uses a lot of knowledge of mathematics. Learners need active learning in mathematics education.

We taught proofs of plane geometry to learners

(University students) by using a method of Groebner basis. Then, learners showed various recognition processes. About recognition process, a lot of results are known in cognitive science. We gave the following problem, and analyzed their solving process.

Problem

Let ABCD be a parallelogram, be the intersection of the diagonals AC and BD. Show AO=OC.



The answer: The implicit assumption that the parallelogram is in a general position means that any three points among the four points A, B, C, and D can be arbitrarily chosen.

Then we can let A=(0,0), B=(u_1,0), C=(u_2,u_3), D=(x_2,x_1), and O=(x_4,x_3).

The hypothesis equations are as follows:

$h_1=u_1x_1-u_1u_3=0$ AB is parallel to DC
$h_2=u_3x_2-(u_2-u_1)x_1=0$ DA is parallel to CB
$h_3=x_1x_4-(x_2-u_1)x_3-u_1x_1=0$ O is on BD
$h_4=u_3x_4-u_2x_3=0$ O is on AC

The conclusion AO=OC is as follows:
$g=2u_2x_4+2u_3x_3-u_3^2-u_2^2=0$.

A proof by using CAS is as follows:.(We use Maple)

```
> with(Groebner):
F := [u1*x1 – u1*u3,  u3*x2 – (u2 – u1)*x1,  x1*x4 – (x2 – u1)*x3 – u1*x1, u3*x4 – u2*x3];
F := [ u1 x1 – u1 u3,  u3 x2 – (u2 – u1) x1,  x1 x4 – (x2 –u1) x3 – u1 x1, u3 x4 – u2 x3]
> G:=Basis(F, plex(x1, x2, x3, x4, u1, u2, u3))
G:= [ - u2 u1 u3 + 2 x4 u1 u3, - u3 x4 + u2 x3,  2 x3 u1 u3 – u1 u3^2,  - u2 u1 u3 + u1 u3 x2 + u1^2 u3,  2 u1 x3 x2 – 2 x3 u1^2 + 2 u1^2 u3 – u2 u1 u3,  x1 u2 – u3 x2 – u1 u3, u1 x1 – u1 u3,  x1 x4 – x3 x2 + x3 u1 – u1 u3]
> factor(G)
[ - u1 u3 ( - 2 x4 + u2 ), - u3 x4 + u2 x3,  - u1 u3 ( - 2 x3 + u3 ),  u1 u3 ( - u2 + x2 + u1 ),  - u1 ( 2 x3 x2 – 2 x3 u1 + 2 u1 u3 – u2 u3 ),  x1 u2 – u3 x2 – u1 u3,  u1 ( x1 –  u3 ),  x1 x4 – x3 x2 + x3 u1 – u1 u3]
> with(Groebner):
 F1 := [u2 - 2*x4,  - u3*x4 + u2*x3,  - 2*x3 + u3, – u2 + x2 + u1, 2*x3*x2 – 2*x3*u1 + 2*u1*u3 – u2*u3,  x1*u2 – u3*x2 – u1*u3, x1 – u3,  x1*x4 – x3*x2 + x3*u1 – u1*u3];
F1 := [ - 2 x4 + u2,  - u3 x4 +u2 x3,  - 2 x3 + u3, – u2 + x2 + u1,  2 x3 x2 – 2 x3 u1 + 2 u1 u3 – u2 u3,  x1 u2 – u3 x2 – u1 u3,  x1 – u3,  x1 x4 – x3 x2 + x3 u1 – u1 u3]
> G1:=Basis(F1, plex(x1, x2, x3, x4, u1, u2, u3))
  G1:= [ - u2 + 2 x4,  2 x3 – u3,  - u2 + x2 + u1, x1 – u3]
> g := 2u2*x4 + 2u3*x3 – u3^2 – u2^2
  g := 2 u2 x4 + 2 u3 x3 – u3^2 – u2^2
> x3=u3/2;
> x4=u2/2;
> g;
> 0
Q.E.D.
```

This problem is a famous problem as introduction using Groebner basis for plane geometry proof.

4. Conclusion

In the three-level model of human behavior, operations and strategies can be identified and considered in relation to human thought processes in order to facilitate error-free problem solving. In consideration of surface features and conditions, similar problems can be recognized and suitable problem-solving methods can be identified. In addition, it was found that contents of the subconscious could be raised to the knowledge -based action level in order to support the expression process and the achievement of efficient functioning.

The technology of theorem prover automated reasoning. The ultimate goal of mathematics is technology. To do mathematics is gaining knowledge and solving problems by reasoning. Theorem prover is a powerful tool for researching mathematics. Researchers should appreciate the possibility of sharing cognitive level with such technology.

When it comes to making students' calculations "activity" through the introduction of technologies such as the CAS, we fear the lack of ability for the calculations. Students can become able to choose by themselves when to use the CAS. In other words, by educating in a way to make the students able to judge the use of the CAS depending on the situation, the worry on the mathematical insight will be avoided.

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Aspects of ICT in Mathematical Activity: Tool and Media

Morten Misfeldt

Abstract—In this article two different approaches to artefacts that support mathematical activity and leaning are investigated; the instrumental approach, which concerns the way that artefacts are made into personal instruments, and the semiotic approach, which concerns the way semiotic representations influence mathematical activity. The motivation for applying these two views of the use of ICT in mathematical activities, is twofold: firstly, computational technology which is used to support mathematical work always involves semiotic representations. Secondly, such representations are used in mathematical activities as a tool that has to be learned and mastered, and which significantly affects solution techniques.

Index Terms— ***E-learning, GeoGebra, Information technology, Instrumental approach, Mathematics ,Semiotics, Writing***

1. *Introduction*

WHEN ICT is used in mathematical activities, two main uses of the technology are sought realized. The first is that ICT can work as a medium for mathematical representations, and thus support both communication and the cognitive mathematical processes that have been shown to rely on external representation (Galison, 2003, DiSessa, 2000, Duval 2006, Winsløw 2003). The second is that ICT works as a tool, and changes various problem situations in mathematics education by allowing easy graphing, algebraic and numerical computations, and visualisations (Dreyfuss 1994, Drijvers & Gravemeijer, 2005, Mariotti 2002, Trouche 2005).

In this article, I address these two uses and the fact that they are often treated as different topics in different theoretical constructs.

In the article, I give a brief and general background for the importance of artefacts for mathematical activities; describe why the semiotic aspects of such artefacts and the basic semiotic units (such as sign and medium) are important in relation to mathematical activities. Furthermore, I describe two theories from mathematics education: (1) Duval's theory of the role of semiotic representations in mathematics education and (2) Truche's instrumental approach. The reason for comparing these two theories is that semiotic representations can act as cognitive tools, and the tools that we use in mathematical activity have semiotic properties which both theoretical frameworks try to address; a comparison will show the strength and weaknesses of each framework. I use these two frames to discuss some examples of ICT use in mathematical activity, researchers' mathematical writing, e-learning at university level and the use of dynamic geometry software.

2. *Internal and External Aspects of Mathematical Activity*

Mathematical activity often relies on an interplay between internal processes, only perceivable by the individual, and external actions, that are also observable by others.

In a Piagetian psychological framework the relation between internal and external processes is expressed as a definition of cognition as an adaptive function developed from – and tested against – empirical reality through actions (Glasersfeld 1995, p. 59). This view of cognition allows for theories of mathematical learning that emphasise relational thinking (Skemp, 1971) and abstraction as a result of particular manipulation with mathematical objects (Dubinsky, 1992). In that sense, mathematical reasoning is grounded in activities external to the mind.

In a sociocultural tradition, the concept 'mediated activity' designates how tools and signs influence and support human activity (Vygotskij, 1978, p. 51). By expanding on Actor Network Theory, Shaffer and Clinton (2006) introduces the concept 'toolforthought' and claim: "In this ontology, then, there are no tools without thinking, and there is no thinking without tools. There are only toolforthoughts, which represent the reciprocal relation between tools and thoughts that exists in both" (p. 291). The concept 'toolforthought' essentially remove the distinction between tools and thoughts, and consider human cognition a matter of working with 'toolforthoughts'.

Semiotic artefacts such as text, diagrams, tables, ect. are important in much knowledge work, not at least in mathematics. Signs and representations on paper can be used to support thinking.

The study of mathematical activity is connected

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to the study of how people interpret and manipulate their environment. In relation to mathematical activity, I consider two important types of 'toolforthoughts' and two theories to describe their use in mathematical activities. The two types are semiotic representations (signs, drawings, diagrams ect.), and the use of computational tools (calculators, Computer Algebra Systems and Dynamic Geometry).

2.1 The instrumental approach

The instrumental approach to the use of technology in mathematics education is developed partly from the discipline 'cognitive ergonomics' (Verillion 1995), and partly from the theory of 'conceptual fields' (Vergnaud 1996).

Verillion and Rabardel works in a sociocultural tradition, end from the assumption that artefacts mediate and shape human agency, the scope of their work is human-computer interaction. Vergnaud is concerned with science and mathematics learning. He is building on the work on Piaget and especially the concept 'scheme': "A scheme is the invariant organization of behaviour (action) for a certain class of situations." (Vergnaud p. 222).

This definition is important, because the instrumental approach is used to examine the process in which the introduction of new technology changes schemes.

The instrumental approach is used to examine how people who use artefacts to address a problem create personal instruments and instrumented techniques to address these problems. Luc Trouche defines an artefact as "a material or abstract object, aiming to sustain human activity" (Trouche, 2005, p. 144). An instrument is what is what the subject builds from the artefact. The process of building an instrument from an artefact is referred to as 'instrumental genesis' and consists of two processes, 'instrumentation' and 'instrumentalisation'. Instrumentation is directed by the subject, towards the artefact. This process includes several phases: discovery, selection, personalisation and transformation. While the first involves getting to know the tool, the latter tends to be a matter of mastering the tool and applying it to one's own very specific needs. Instrumentalisation is directed by the artefact towards the subject. It is the process in which the subject adapts to the new opportunities and constraints the tools offer. In the instrumentation process, the tool shapes the behaviour of the subject confronted with a specific type of tasks.

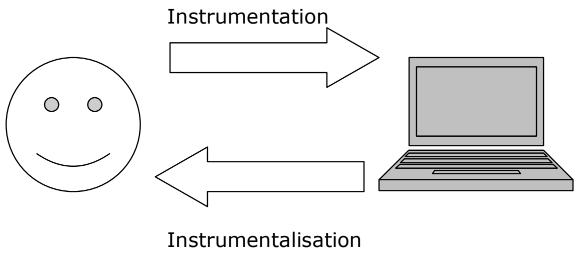

Figure 1: The instrumental approach to the use of computers for mathematics, describes the interplay between person and artefact as a bi-directional process consisting of instrumentation, directed towards the artefact, and instrumentalisation, directed towards the person.

2.2 The semiotic approach

Mathematical discourse is rich in symbols, formulas and diagrams, and Raymond Duval has developed a theoretical machinery to describe the potentials and pitfalls of working with many representations of abstract mathematical concepts (Duval, 2006).

A semiotic representation or 'sign' consists of a material signifier which stands for something: the signified. The material substance that is manipulated in order to create representations is denoted 'medium'. Examples of media are pen and paper and a computer with a specific configuration of programs.

The central aspect of the theory is transformations of semiotic representations, particularly treatments and conversions (see figure 2). Treatments are transformations inside a semiotic system, such as rephrasing a sentence or isolating x in an equation. Conversion is a transformation that changes the system, maintaining the same conceptual reference, such as going from an algebraic to a geometric representation of a line in the plane.

Mathematical objects are typically not there to be pointed at as anything but representations, sometimes even representations of a very technical nature, and mathematical objects always have more than one semiotic representation attached to it (Duval, 2006). These two facts lead to two fundamental issues in learning mathematics; (1) one common mistake is to confuse the mathematical object with one of its representations, and (2) transformation of semiotic representations can be difficult, but a lot of the creative potential in mathematics stem from transformations of semiotic representations (e.g. calculations) (Duval 2006).

Duval shows that students often have problems with changing between types of representation, particularly if this change of representational form not include a recipe for translating parts of representation in the starting register to parts of the representation in the target register. One example of a problematic change in representational form is from a plot of a function to an algebraic formulae, whereas the other way (from formula to graph) is conceptually simple since creating a table of (x,f(x)) values in principle constitute a recipe.

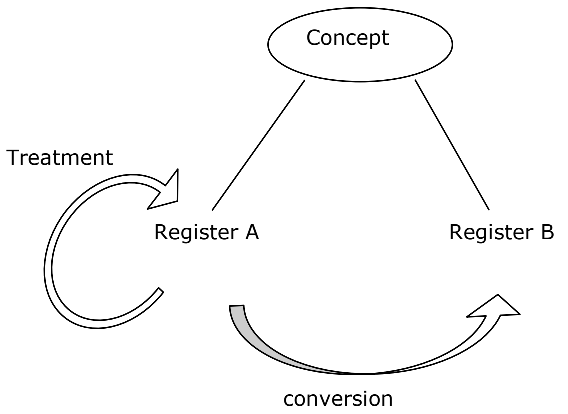

Figure 2: The transformative processes: conversion and treatment.

Conceptual understanding can be described as a person's degree of freedom in choosing various semiotic representations of the same mathematical concept (Winsløw 2003).

3. EXAMPLES OF MATHEMATICAL ACTIVITY

Representations, media and tools can affect mathematical activity. In various mathematical practices, this interplay can take different forms. Below, I describe some examples that show aspects of the relation between representations, media and tools. The first example concerns mathematicians' use of various media to support their writing process. The second is e-learning based teaching of undergraduate mathematics, and the final example is the use of dynamic geometry software in teaching of mathematics.

3.1 Researchers' mathematical writing

I have conducted an interview study among professional mathematicians (Misfeldt, 2006). The objective of the investigation was to understand the writing process of professional mathematicians from early idea to finished paper. In particular, I compared the purposes that writing served with the mathematician to the types of representations he/she used and the media (computer or pen and paper) he/she chose to use.

The result of the investigation suggests that it makes sense to consider the following five different functions of writing in mathematics (Misfeldt 2006):

Heuristic treatment consisting of getting and trying out ideas and identifying connections.

Control treatment is a deeper investigation of the heuristic ideas. It can have the form of pure control of a proposition or an open-ended investigation e.g. a calculation. It is characterised by precision.

Information storage to save information for later retrieval. Either electronically or on paper.

Communication with fellow collaborators: Ranges from annotation of an existing text over comments or ideas concerning a collaborative project to suggestions of sections to add to a paper.

Production of a paper, where writing is used to deliver a finished product intended for publication and aimed at a specific audience.

The respondents showed a strong tendency to use visual and diagrammatic types of semiotic representations to support the functions of heuristic treatment, and to some extend control treatment.

Pen and paper was dominant in the respondents' choice of medium to support heuristic and control treatment, and for some of the respondents also to support information storage. Paper is superior for heuristic treatment for two reasons. Paper is a very user-friendly technology with respect to start-up time, (screen) real estate and portability (Sellen and Harper, 2002), and paper supports a multimodal form of expression were a number of mathematical registers can be activated simultaneously (Misfeldt, 2008).

3.2 E-learning in mathematics: remediation of a flexible medium

The initiative Delta, matematik på nett, at the Norwegian technical university, consists of eight online courses in undergraduate mathematics. The topics include: Calculus, linear algebra, geometry, number theory, probability and statistics. The primary platform for the program is the learning management system 'Moodle'. The course material consists of a syllabus, brief video lectures and mandatory exercises.

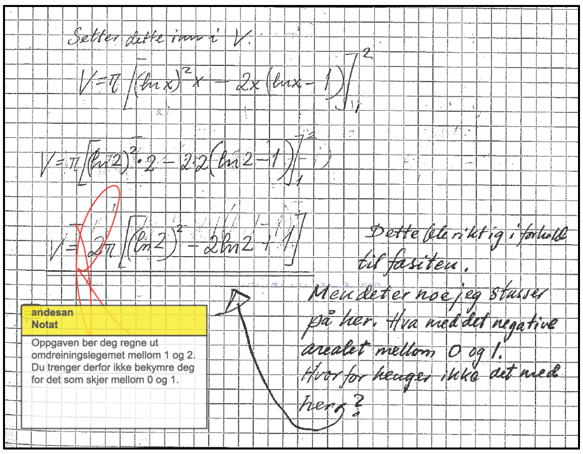

Figure 3: Teacher-student communication is mediated by handwriting.

The communication between teacher and learners about the written assignments is mediated by handwriting. The students are required to buy a scanner and scan their handwritten manuscripts in order to upload them to the learning management system, where it is reviewed and commented by the teacher, who uses an electronic pen (Misfeldt & Sanne 2007). The teachers find that this way of communication best meets their needs. The learning management system provides a discussion forum for the students', here, keyboard generated text is used to discuss mathematics. The support for mathematical notation in the discussion forum is considered insufficient by the students so they either use basic keyboard notation (such as f(x)=(3+x)/(2-x)) or tries to cut and paste formulas from other applications (Misfeldt and Sanne 2007).

3.3 Dynamical geometry: a tool that provides dynamic representations

Dynamic geometry software such as GeoGebra provides a different, more interactive and theory-related type of mathematical diagram than paper allows (Larborde, 2005). Furthermore, GeoGebra allows for a close connection between the symbolic manipulation and visualisation capabilities of computer algebra system (CAS) and the dynamic abilities of dynamic geometry software (DGS). The user can work with points, vectors, segments, lines and conic sections, but equations and coordinates can also be entered directly, and functions can be defined algebraically and then changed dynamically (Hohenwarter & Jones, 2007).

One example of GeoGebra as a "vehicle for learning" is that the concept of derivative, can be taught without the use of limits (Andresen & Misfeldt 2010).

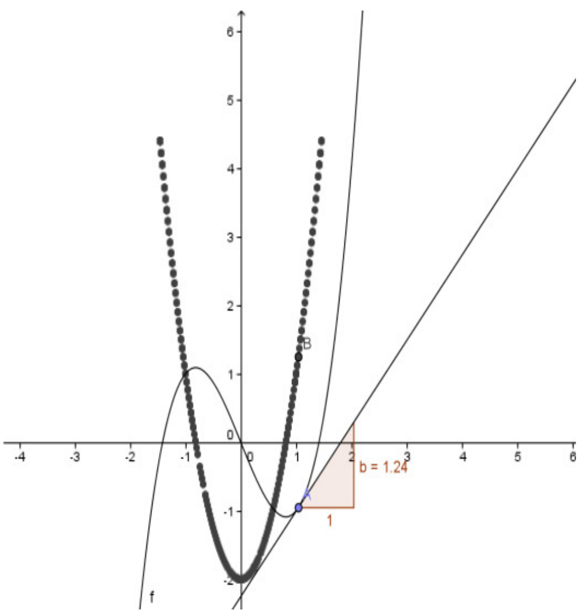

Figure 4: Teaching derivatives without limits.

In the above example, we see one way that ICT can allow a special kind of interactive representation when the teacher introduces the notion of 'derivative'. The above drawing was constructed by placing a point A on the graph of the function, and then apply the geometrical tangent to the graph of f. GeoGebra allows you to move the point around on the graph. You can then place a geometrical tangent to the graph through the point A. By defining a point B with the x-coordinate from point A and y-coordinate that is the slope of the tangent through A you can get the shape of the graph of the derivative function without considering limits at all. The example shows that the dynamic representations that the tool GeoGebra provides changes the dynamics between the geometric and algebraic register when we work with derivatives.

4. DISCUSSION

This discussion revolves around three themes: (1) Instrumental aspects of semiotic representations, (2) instrumental aspects of media, and (3) semiotic aspects of mathematical tools.

4.1 Instrumental aspects of semiotic representations

We can view semiotic representations as a specific class of artefacts that can be turned into instruments by a process of instrumental genesis. Duval (2006) shows that transformation of semiotic representations is one example of an instrumented technique applied to solve a mathematical problem. In this case, the instrument is the semiotic representations, and the development of such instrumented techniques can potentially lead to learning problems. Furthermore if the formation of viable mathematical concepts are connected to the ability to coordinate various representations of the same concept without favouring one of them, it is a relevant hypothesis that expanding of the number of representations of a concept may improve conceptual understanding.

In the example with the mathematicians' writing process, it is clear that some of the more visual and diagrammatic uses of representations belongs in the creative phase of mathematical work rather than in the communicating phase. This is an indication that these visual representations are used as mathematical instruments in a problem-solving process.

4.2 Instrumental aspects of media

In the mathematical e-learning project described, we saw how the dependence on various forms of semiotic representations affected the choice of media. They used handwritten drafts (scanned to pdf files), even though this is an unusual way to communicate electronically. This remediation was a pragmatic attempt to make the computer support handwriting, and in that sense it is a process of instrumentation, where the computer medium is controlled by the mathematical user and adapted in order to fit his/her needs. The cognitive instrument developed through the instrumental genesis is in many ways similar to handwriting. This result is interesting from an instrumental point of view, because one plausible reason for developing an instrument of 'remediated handwriting' is that handwriting has certain properties that makes it useful as a mathematical instrument.

That handwriting has some properties that allows it to serve as a mathematical instrument is also verified in the interviews with mathematicians. The mathematicians demonstrate a transition from handwriting in the early/creative part of the work towards the use of computers for information storage and communication. Again this implies that there are some aspects of pen and paper that allows it to develop into a mathematical instrument.

4.3 Semiotic aspects of mathematical instruments

To study semiotic aspects of instruments means to study how tools affect knowledge representation and designation of meaning.

The close connection between algebraic and geometric representation in GeoGebra can be described with Duval's framework. GeoGebra provides a constant double representation of mathematical concepts. This means that conversions could very well be rather different, cognitively, when you do geometry in GeoGebra, compared to paper and pencil. According to Duval, one of the most difficult aspects of learning mathematics is to learn to handle conversion between registers. However the constant presence of the two most important registers makes a potential large difference in accessibility to the mathematical topic of analytic geometry, in the "representational competence" that working with the topic requires and hence (maybe only hypothetically) in the relevance of the topic as an engine to promote general literacy. The key factor for why Geogebra is a unique piece of software lies in its semiotic abilities, for instance in the simultaneous and dynamic representation of multiple registers.

5. CONCLUSION

In this paper, I have described how computational tools and representations support mathematical activities and learning. The basic claim is that it makes sense to study (1) the semiotic aspects of mathematical tools (e.g. computational tools and physical manipulatives), (2) the instrumental aspects of semiotic representations, i.e. what kinds of instrumented techniques a specific register can allow a user to develop, and (3) the instrumental aspects of the used media, for example by looking at what types of registers, and conversions between registers, a specific media facilitates. Furthermore it makes sense to study mathematical activities that rely on uses of media and representations, as a process of developing instrumented techniques, through instrumental genesis where the mathematical worker/learner on one side changes her approach to a mathematical challenge because of she is influenced by the possibilities and constraints that media and representations provides and on the other hand takes control over the representation and media that she works with and changes it to fit personal needs.

The potential of combining a semiotic and an instrumental approach is that it can allow for at description of the way toolforthoughts (Shaffer & Clinton 2006) are used as an integrated part of mathematical activity. Representations, media and tools as disjoint aspects of mathematical activities do not tell the full story. This paper represent an attempt to combine these aspects.

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Remember

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The Xmath Partial Differentiation algorithm

Odd Bringslid

Abstract—The Xmath eBook is being developed and algorithms into a wide range of undergraduate mathematical issues embeded in Mathematica packages are available on the web using the system webMathematica. The main purpose is to visualize mathematics in the same way as would a professor do it on the blackboard stating all intermediate steps for user defined input and then presenting solutions being easily recognized by the undegraduate student which may not always be the case using the Mathematica system directly. In this way the student may work more on a personal basis, viewing one step at a time in the solving process and then being less dependent of the professors. In this paper The Xmath Algorithm for Partial Differentiation step-by-step are presented (PD Steplet)

Index Terms— Partial Differentiation (PD), Steplet, Mathematica packages, Online calculations, Pedagogical value

1. *Introduction*

The use of *Mathematica* [1] in education is one of the most important areas of application. The problem however is that in education we are focusing on how problems are solved perhaps more than on the final result. Since *Mathematica* only gives the final result it will be necessary to build an application on top of *Mathematica* giving intermediate results using the methods of solving given by mathematical textbooks. It is necessary to analyze the equations in depth, Xmath then using the *Mathematica* object TreeForm to be able to extract the information needed at each level of the solution process. The algorithm is different from the algorithm used by the developers of the *Mathematica* System (D).

The Xmath algorithm will solve problems typical in mathematical teaching. General partial differentiation is implemented using standard methods, tracking the solving process in detail to be easily recognized by the students.

2. *Pedagogical value*

The pedagogical value of the Xmath algorithms lies in the fact that a student may simulate solving by changing parameters and type of function. The important thing is that Xmath solves the equations as would a professor do it on the blackboard then easily being recognized by the students which is not the case using the *Mathematica* system directly [2].

3. *Example*

Level 1

Find the partial derivative $\frac{\partial}{\partial y}\left(\frac{\sin(x\cdot y)}{x^2+y^2}\right)$

Quotient Rule

$$\frac{\partial \frac{u}{v}}{\partial y} = \frac{v\frac{\partial u}{\partial y} - u\frac{\partial v}{\partial y}}{v^2}$$

$Here\ u=\sin(x\cdot y)\ and\ v=x^2+y^2,\ Finding\ derivatives\ of\ u\ and\ v$

Level 2

Find the partial derivative $\frac{\partial \sin(x\cdot y)}{\partial y}$

Chain Rule, Composite function

$Substitute\ u=g(y)$

$$\frac{\partial f(g(y))}{\partial y} = \frac{\partial f}{\partial u}\cdot\frac{\partial u}{\partial y}$$

$Inside\ \ function\ u=x\cdot y\ and\ outside\ \ f(u)=\sin(u)$

Level 3

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Find the partial derivative $\frac{\partial(x\cdot y)}{\partial y}$

Linear Rule, Constant factor

$$\frac{\partial(c\cdot f)}{\partial y}=c\cdot\frac{\partial f}{\partial y},\ \textit{Here } c=x \textit{ and } f(y)=y$$

Finding the derivative of the non−constant factor $f(y)=$

Level 4

Find the partial derivative $\frac{\partial y}{\partial y}$

Power Rule

$$\frac{\partial y^n}{\partial y}=n\cdot y^{n-1},\ \textit{Here } n=1$$

$$\frac{\partial y}{\partial y}=1$$

Result, Linear Rule Constant factor

Level 3

Find the partial derivative $\frac{\partial\sin(u)}{\partial u}$

Sin Rule

$$\frac{\partial\sin(u)}{\partial u}=\cos(u)$$

$$\frac{\partial f}{\partial u}=\cos(u)$$

Result, Chain Rule

Substitute $u=g(y)=x\cdot y$

$$\frac{\partial\sin(x\cdot y)}{\partial y}=\frac{\partial(g(y))}{\partial y}=\frac{\partial f}{\partial u}\frac{\partial u}{\partial y}=x\cdot\cos(x\cdot y)$$

This gives $\frac{\partial u}{\partial y}=x\cdot\cos(x\cdot y)$ *and* $v\frac{\partial u}{\partial y}=x(x^2+y^2)\cos(x\cdot y)$

Level 2

Find the partial derivative $\frac{\partial(x^2+y^2)}{\partial y}$

Linear Rule, Sum

$$\frac{\partial(x^2+y^2)}{\partial y}=\frac{\partial x^2}{\partial y}+\frac{\partial y^2}{\partial y}$$

Level 3

Find the partial derivative $\frac{\partial x^2}{\partial y}$

Constant Rule

Derivative of a constant is 0 (independent of y)

$$\frac{\partial x^2}{\partial y}=0$$

Result, Linear Rule Sum

$$\frac{\partial(x^2+y^2)}{\partial y}=\frac{\partial y^2}{\partial y}+\frac{\partial x^2}{\partial y}=2y$$

This gives $\frac{\partial v}{\partial y}=2y$ *and* $u\frac{\partial v}{\partial y}=2y\sin(x\cdot y)$

in the second part of numerator of the rule.

We then find the numerator:

$$v\frac{\partial u}{\partial y}-u\frac{\partial v}{\partial y}=x(x^2+y^2)\cos(x\cdot y)-2y\cdot\sin(x\cdot y)$$

Result, Quotient Rule (Answer)

$$\frac{\partial\frac{\sin(x\cdot y)}{x^2+y^2}}{\partial y}=\frac{v\frac{\partial u}{\partial y}-u\frac{\partial v}{\partial y}}{v^2}=$$

$$\frac{x(x^2+y^2)\cos(x\cdot y)-2y\sin(x\cdot y)}{(x^2+y^2)^2}$$

4. THE LINEAR RULE

The linear rule will differentiate a function like

(1) $f(x_1,x_2,..)=a_1f_1(x_1,x_2,..)+a_2f_2(x_1,x_2,..)+...$

Here a_i is independent of x. The rule is divided into the linear rule sum and the linear rule consant factor.

The function is broken down for analyzing by using the *Mathematica* object TreeForm [3] with 2 levels. The *Mathematica* object D [4] gives the derivative. Linear rule sum:

```
LinearList=Reverse[Level[TreeForm[f],2]];
LinearList=Delete[LinearList,1];
Do[Main[ak*yk,level],{j,1,Length[LinearList]}]
] (*end Do*));
Result=Sum[Main[ak*yk]]
```

Figure 1 PseudoCode Linear Rule Sum

```
FactList=Reverse[
  Level[TreeForm[a*y[x]],2]];
If [ FreeQ[FactList[[3]],x],
Result=FactList[[3]] * Main[FactList[[2]],x,level]]
```

Figure 2 PseudoCode Linear Rule Constant Factor

5. The Quotient Rule

The quotient rule will differentiate a function like

$$(2) \quad f(x_1, x_2, ...) = \frac{u(x_1, x_2, ...)}{v(x_1, x_2, ...)}$$

```
expr=u/v;
  u=Numerator[expr];
  v=Denominator[expr];
Result=Main[D[u,x]*v-D[v.x]*u)/v^2,level];
```

Figure 3 PseudoCode QutientRule

6. Chain Rule

This rule is used for a function being a composite function of the form y=f (g(x)). The derivative is given by the chain rule

$$u = g(x)$$
$$\frac{\partial f(g(x))}{\partial x} = \frac{\partial f(u)}{\partial u} \cdot \frac{\partial u}{\partial x}$$

```
expr=g[u[x]];
u=Reverse[Level[TreeForm[g[u[x]], 2]];
Result=Main[D[expD[g[u],u]*D[u[x]];
```

Figure 4 PseudoCode ChainRule

Main Program

PseudoCode given for rules used in the eaxmple.

```
Main[expr,x,level]:=Module[{},
   k=Reverse[Level[TreeForm[e …)xpr,2];
   Which[
      FreeQ[expr,x],DConstantRule[expr,x,level],
          expr===x,DxRule[expr,x,level],
          Head[expr]===Tan,
If[Last[k]===x,DTanRule[expr,x,level],DChainRule[expr,x,level
]],
          (* Same system for
Cos,Sin,Log,Arctan,ArcSin,ArcCos*)
     Head[expr]===Plus,DPlusRule[expr,x,level]
          Head[expr]===Times&&(Not[FreeQ[Denominator[
k[[2]]],x]]
                  v Not[FreeQ[Denominator[k[[3]]],x]]),
DQuotRule[expr,x,level],
          Head[expr]===Times&&FreeQ[Last[k],x] v
          FreeQ[k[[2]],x],DProdCoRule[expr,x,level],
     Head[expr[===Times, DProductRule[expr,x,level],

Head[expr]===Power&&k[[3]]===x&&FreeQ[k[[2]],x],DPower
Rule[expr,x,level],

Head[expr]===Power&&FreeQ[k[[2]],x],DChainRule[expr,x,lev
el],

Head[expr]===Power&&Head[k[[2]]]===Symbol&&FreeQ[k[[3]
],x],
          DDPowerRuleExp[expr,x,level],

FreeQ[k[[3]],x]&&Head[expr]=Power,DChainRuleExp[expr,x,le
vel],
   Head[expr]==Power&&Not[FreeQ[k[[2]],x]],
   LogarithmicRule[expr,x,level]
 ]
```

Figure 5 PseudoCode Main Program

The Main Program invokes the Mathemaica objects FreeQ [5], Head[6], Which[7] and Reverse [8]

Acknowledgment

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